

Student Workbook

Polar Form of Complex Numbers

Learning Target: I can convert complex numbers to/from polar form, multiply and divide them in polar form, and apply De Moivre's Theorem.

Vocabulary

Term	Meaning
Modulus (r)	The distance from the origin: $r = \sqrt{a^2+b^2}$.
Argument (θ)	The angle the complex number makes with the positive real axis.
De Moivre's Theorem	$z^n = r^n(\cos(n\theta) + i\sin(n\theta))$

Key Concept: Converting to Polar Form

EXAMPLE 1

Convert $3+3i$ to polar form.

$$r = \sqrt{9+9} = \underline{\hspace{2cm}} \quad \theta = \underline{\hspace{2cm}} \rightarrow z = \underline{\hspace{4cm}}$$

Key Concept: Converting to Rectangular Form

EXAMPLE 2

Convert $2(\cos 60^\circ + i\sin 60^\circ)$ to rectangular form.

$$a = 2\cos 60^\circ = \underline{\hspace{2cm}} \quad b = 2\sin 60^\circ = \underline{\hspace{2cm}}$$

Key Concept: Multiplying & Dividing in Polar Form

MULTIPLY: multiply r 's, **ADD** angles **DIVIDE:** divide r 's, **SUBTRACT** angles

EXAMPLE 3

Multiply $z_1=2(\cos 30^\circ + i\sin 30^\circ)$ and $z_2=3(\cos 45^\circ + i\sin 45^\circ)$.

Product = $(2 \cdot 3)(\cos(30^\circ + 45^\circ) + i\sin(30^\circ + 45^\circ)) =$ _____

COMMON MISTAKE

In De Moivre's Theorem, the modulus r gets raised to the POWER n (r^n), but the argument θ gets MULTIPLIED by n ($n\theta$) — these are two different operations. Don't multiply r by n !

Key Concept: De Moivre's Theorem

EXAMPLE 4

Use De Moivre's Theorem to find $(1+i)^4$.

$1+i$: $r=\sqrt{2}$, $\theta=45^\circ$

$(1+i)^4 = (\sqrt{2})^4(\cos(4 \cdot 45^\circ) + i\sin(4 \cdot 45^\circ)) =$ _____ $(\cos 180^\circ + i\sin 180^\circ) =$ _____

Guided Practice — Part A: Convert to Polar Form

A1 $5+5i$

A2 $-4+4\sqrt{3}i$

Guided Practice — Part B: Convert to Rectangular Form

B1 $4(\cos 120^\circ + i\sin 120^\circ)$

B2 $6(\cos 300^\circ + i\sin 300^\circ)$

Guided Practice — Part C: Multiply & Divide

C1 Multiply $3(\cos 40^\circ + i\sin 40^\circ)$ and $2(\cos 20^\circ + i\sin 20^\circ)$.

C2 Divide $10(\cos 80^\circ + i\sin 80^\circ)$ by $2(\cos 30^\circ + i\sin 30^\circ)$.

Guided Practice — Part D: De Moivre's Theorem

D1 Find $(1+i\sqrt{3})^3$.

D2 Find $(\sqrt{3}-i)^4$.

BEFORE YOU MOVE ON

Check with a partner: in De Moivre's Theorem, did you raise r to the power n , and MULTIPLY (not raise) θ by n ?